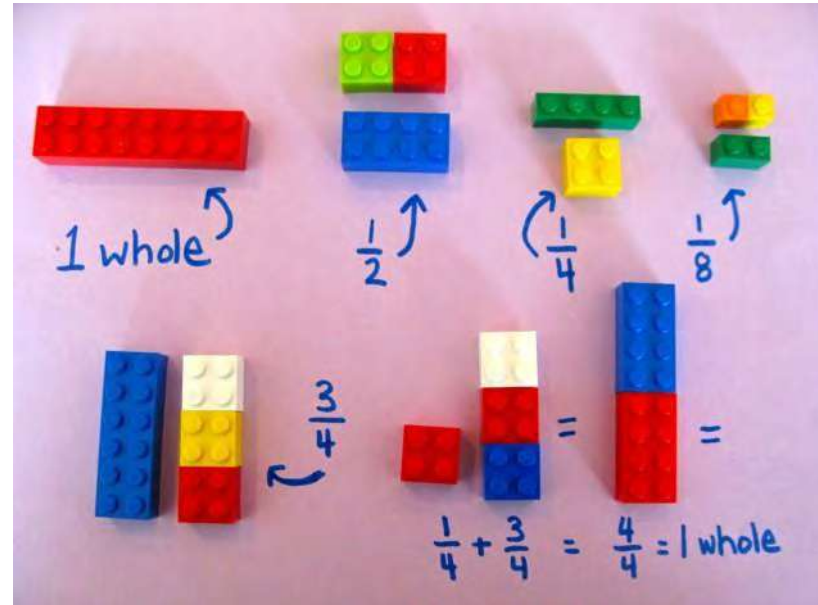


Ocklynge Junior School



Progression in Fractions

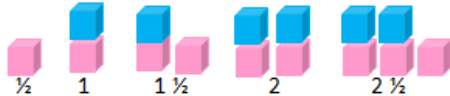
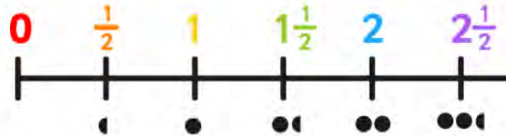
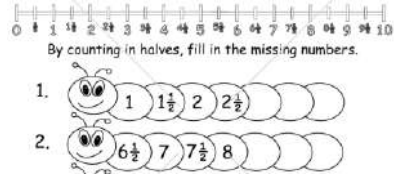
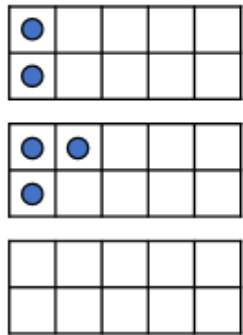
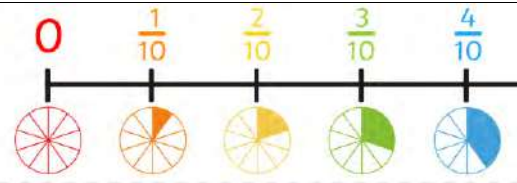
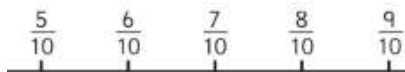
Written by

Miss Shepherd and Mrs Quine

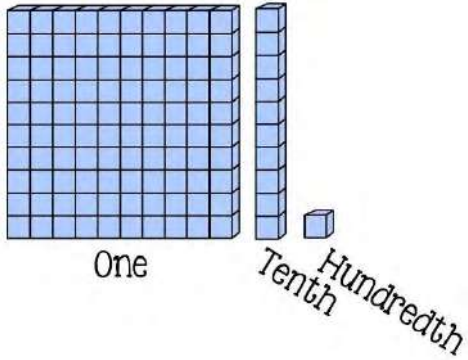
Key vocabulary when teaching fractions

Word	Definition	Example
Fraction	A part of a whole number, quantity or shape. 2. Expressing a division relationship between two integers in the form $\frac{a}{b}$	I have shared my sweets into four equal parts. Everyone will get a fraction of the whole quantity of sweets. One group is a quarter of the whole
Numerator	The number written above the fraction line in a fraction. It indicates the specified number of parts out of the whole. In a division context, it is the dividend.	In the fraction one quarter, one is the numerator.
Denominator	The number written below the fraction line in a fraction. In a measure context, it indicates the number of equal parts into which the whole is divided. In a division context, it is the divisor.	In the fraction one quarter, four is the denominator.
Unit fraction	A fraction with a numerator of one.	One-third is a unit fraction.
Non-unit fraction	A fraction with a numerator greater than one.	Two thirds is a non-unit fraction.
Equivalent	Equivalent means having the same value. Equivalent fractions have the same value.	$\frac{2}{4} = \frac{1}{2}$
Proper fraction	A fraction with a value less than one.	$\frac{1}{2}, \frac{3}{4}, \frac{5}{8}$
Improper fraction	A fraction where the numerator is bigger than the denominator. These fractions are therefore greater than one whole.	$\frac{12}{11}$
Mixed numbers	Numbers consisting of an integer and fractional part.	$1\frac{1}{2}; 3\frac{3}{4}$
Decimal fraction	A fraction expressed in its decimal form.	Half written as a decimal fraction is 0.5
Proportion	A comparison between two or more parts of a whole or group. Proportion expresses a part-whole relationship. This may be represented as a fraction, a percentage or a decimal.	Two thirds of a class were boys. The proportion of the class that is girls is one third.

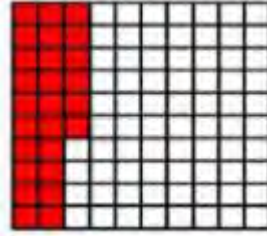
Counting in fractional steps

	Objectives	Concrete	Pictorial	Abstract	Challenges
Stage 2	<p>Pupils should count in fractions up to 10, starting from any number and using the $\frac{1}{2}$ and $\frac{2}{4}$ equivalence on the number line.</p>	 <p style="text-align: center;">How many halves in?</p>		 <p>By counting in halves, fill in the missing numbers.</p>	<p>Spot the mistake $7, 7 \frac{1}{2}, 8, 9, 10, 8 \frac{1}{2}, 8, 7, 6 \frac{1}{2}, \dots$ and correct it</p> <p>What comes next? $5 \frac{1}{2}, 6 \frac{1}{2}, 7 \frac{1}{2}, \dots, \dots$ $9 \frac{1}{2}, 9, 8 \frac{1}{2}, \dots, \dots$</p>
Stage 3	<p>Count up and down in tenths</p>	<p>Lucie is using counters to show tenths.</p>  <p>Using tens frames and counters, show the next tenth in the sequence.</p>	 <p>Draw the next tenth in the sequence.</p>		<div style="border: 1px solid blue; padding: 5px; margin-bottom: 10px;"> <p>I count backwards four tenths. My answer is $\frac{10}{10}$. What fraction did I start with?</p> </div> <p>What comes next? $\frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \dots,$ $\dots, \frac{12}{10}, \frac{11}{10}, \dots,$ \dots, \dots</p>

Count up and down in hundredths



Using base 10, children can show counting in hundredths. Base 10 can also be used to show the link between tenths and hundredths.

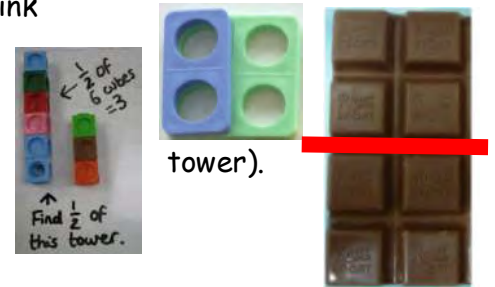
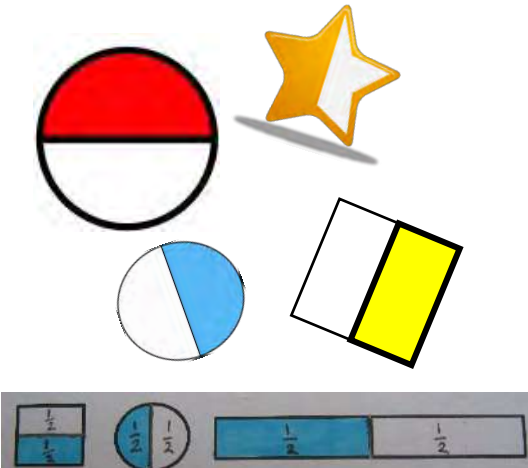
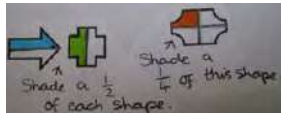
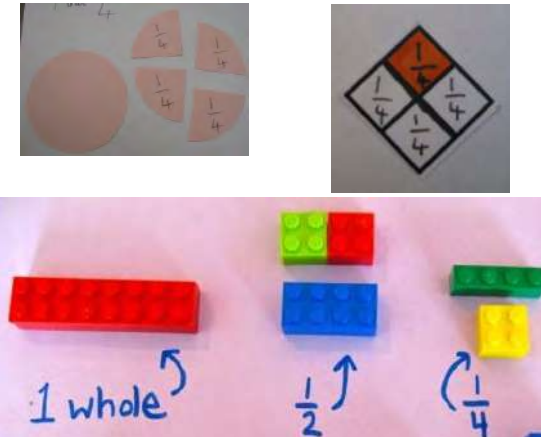
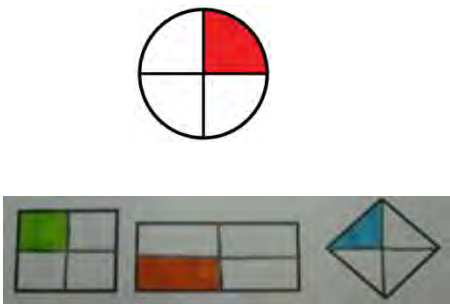
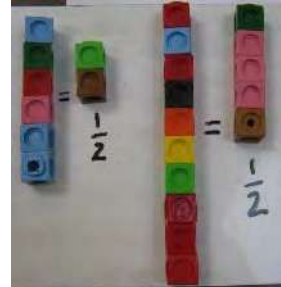


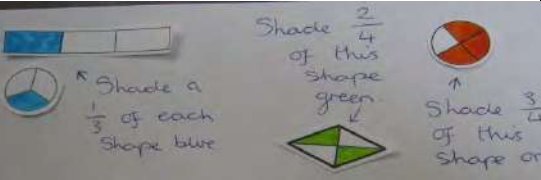
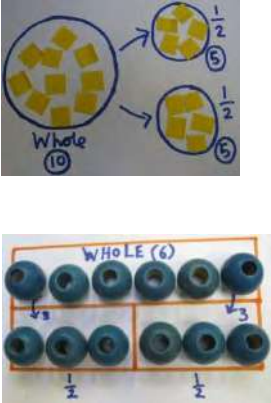


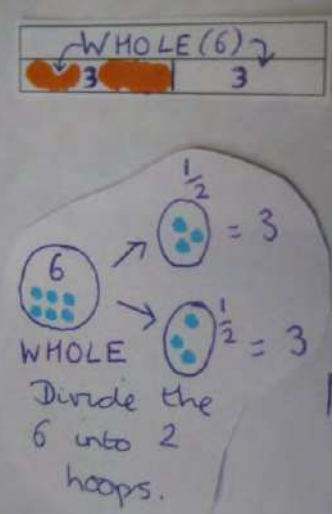

Use a hundred square to shade in a sequence of hundredths.

As a class, count up and down in hundredths. Children to continue the following pattern:
 $1 \div 100 = \frac{1}{100}$, $2 \div 100 = \frac{2}{100}$,
 What do they notice? What is a hundredths? How many hundredths make up a whole?

Spot the mistake
 sixty hundredths,
 seventy hundredths,
 eighty hundredths,
 ninety hundredths,
 eleven hundredths ...
 and correct it.

Recognising fractions

	Objectives	Concrete	Pictorial	Abstract	Challenges
Stage 1	Recognise, find and name a half as one of two equal parts of an object, shape or quantity.	<p>Cutting up objects (fruit, paper shapes, jaffa cakes) into halves and quarters. Finding a half and a quarter of an object (smarties, beads, multi-link tower).</p> 	<p>Different representations of a half:</p> 	<p>$\frac{1}{2}$ One half</p>	<p>Shading fractions of a range of shapes.</p> 
Stages 1	Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.	<p>Cutting up objects into quarters. Finding a quarter of an object (smarties, beads, multi-link tower).</p> 		<p>$\frac{1}{4}$ One quarter</p>	<p>Identifying halves and quarters from different representations.</p> 

<p>Stage 2</p>	<p>Recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity</p>			<p>$\frac{1}{2}$ of 6 cubes = 3 $6 \div 2 = 3$</p>	 <p>Which of these shapes are $\frac{1}{2}$ green?</p>
<p>Stage 2</p>	<p>Write simple fractions for example, $\frac{1}{2}$ of 6 = 3 and recognise the equivalence of $\frac{1}{2}$ and $\frac{2}{4}$.</p>	<p>Finding a $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ set of objects (less than 20 objects).</p> <p>$\frac{1}{3}$ of 9 cubes = 3</p> 	<p>$\frac{1}{2}$ of 6 cubes = 3</p> 	<p>$\frac{1}{2}$ of 6 cubes = 3 $6 \div 2 = 3$</p>	 <p>Leo lost $\frac{1}{2}$ his marbles in a game. This is what he has left. How many did he start with?</p>

Recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators.



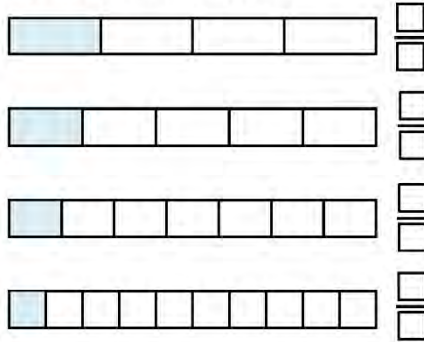
What fractions are being represented by the Numicon?

What different fractions can you represent with these classroom objects?

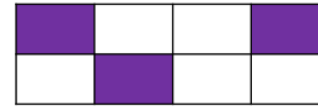


2/3 of the scissors are red.

What fraction of each bar is shaded?



Circle the fraction represented here.



$\frac{8}{5}$ $\frac{3}{8}$ $\frac{3}{5}$

Read the picture

What fraction of the shape is red?
What fraction of the shape is blue?



Four out of twelve equal parts

$$\frac{4}{7}$$

Two thirds

$$\frac{3}{4}$$

A	
B	
C	
D	

Match the fraction with its representation.

Challenge 1:

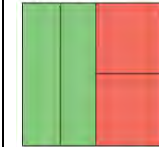
What fractions of the whole is coloured?


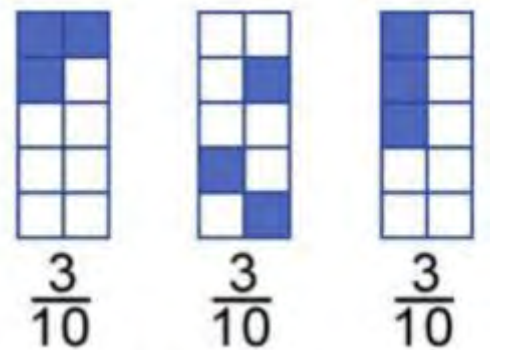
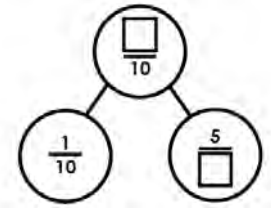
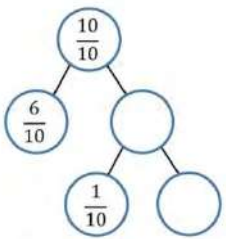
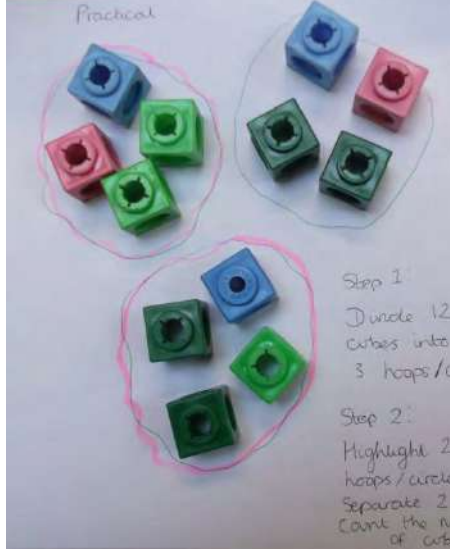
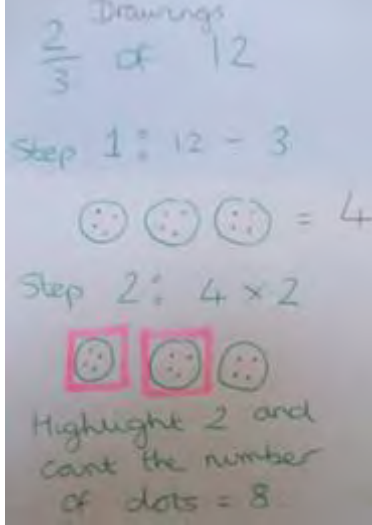



Challenge 2:

The shape is divided into 4 equal parts. Do you agree?

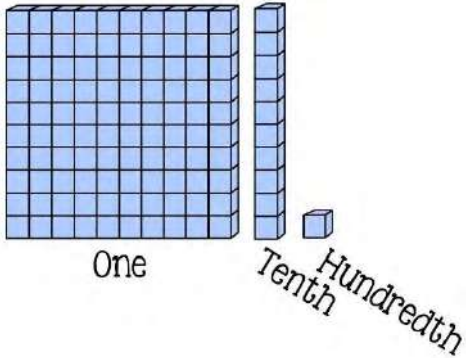
Explain why.



<p style="writing-mode: vertical-rl; transform: rotate(180deg);">Stage 3</p>	<p>Recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10.</p>	<p>Any combination of ten objects can be used to represent tenths.</p>  <p>There are ten sweets in a packet. Five of them are striped. Write the number of stripy sweets as a fraction. Counters and tens frames can also be used to represent tenths.</p>	 <p style="text-align: center;">$\frac{3}{10}$ $\frac{3}{10}$ $\frac{3}{10}$</p>	 <p>Complete the part whole model.</p> $\frac{5}{10} + \underline{\quad} = \frac{10}{10}$	<p>Fill in the missing values. Explain how you got your answers.</p> 
<p style="writing-mode: vertical-rl; transform: rotate(180deg);">Stage 3</p>	<p>Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators</p>	 <p><i>Practical</i></p> <p>Step 1: Divide 12 cubes into 3 heaps/circles</p> <p>Step 2: Highlight 2 heaps/circles - Separate 2 Count the number of cubes</p>	 <p><i>Drawings</i></p> <p>$\frac{2}{3}$ of 12</p> <p>Step 1: $12 \div 3 = 4$</p> <p>$4 \times 2 = 8$</p> <p>Highlight 2 and count the number of dots = 8</p>	<p>$\frac{1}{5}$ of 15 sweets = 3 as $15 \div 5 = 3$</p> <p>$\frac{3}{5}$ of 15 sweets = 9 as $15 \div 5 = 3$ and $3 \times 3 = 9$</p>	<p>True or false?</p> <p>$\frac{2}{10}$ of 20cm = 2cm $\frac{4}{10}$ of 40cm = 4cm $\frac{3}{5}$ of 20cm = 12cm</p>  <p>This is $\frac{2}{5}$ of a bag of marbles. How many marbles are in a full bag?</p>

Stage 4

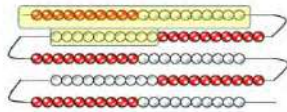
Recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten



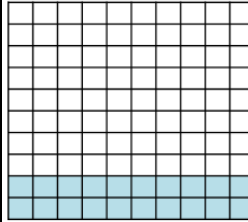
Using base 10, children can show hundredths.

Base 10 can also be used to show the link between tenths and hundredths.

If the whole bead string represents one whole, what decimal is represented by the highlighted part? Can you represent this on a 100 square?



Which of the following statements are correct?



- 20 hundredths is equivalent to 2 tenths.
- 2 hundredths is equivalent to 20 tenths.

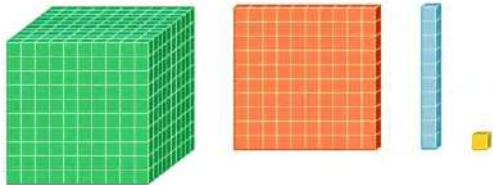
$\frac{1}{10}$ of 60 = 0.6 because
 $60 \div 100 = 0.6$
 $\frac{1}{10}$ of 70 = 0.7 so $\frac{1}{100}$ of 70 = 0.07



What hundredths are the arrows pointing to?

Stage 5

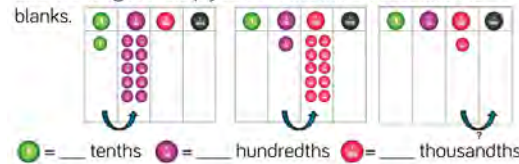
Recognise and use thousandths and relate them to tenths and hundredths.



Using base 10 with the thousand cube as a whole, children can show thousandths.

Base 10 can also be used to show the link between tenths, hundredths and thousandths.

Use the images to help you fill in the third model and the blanks.



● = ___ tenths ● = ___ hundredths ● = ___ thousandths

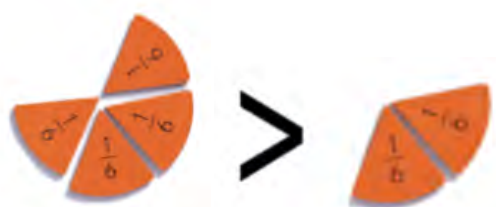

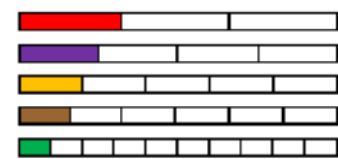
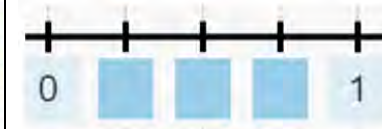

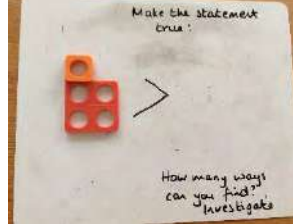
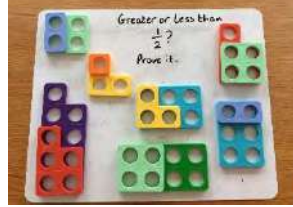


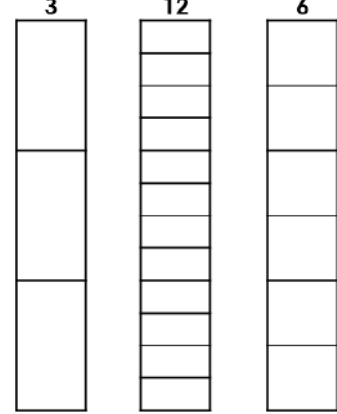
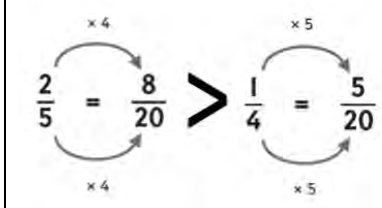
$$\frac{7}{10} = \frac{\quad}{100} = \frac{\quad}{1000}$$

What do you notice?

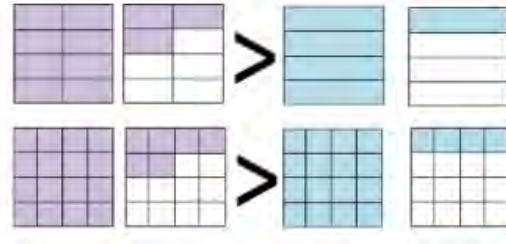
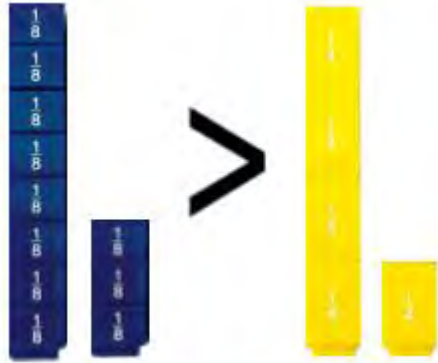
One tenth of £41
 One hundredth of £41
 One thousandth of £41

Continue the pattern
 What do you notice?
 $0.085 + 0.015 = 0.1$
 $0.075 + 0.025 = 0.1$
 $0.065 + 0.035 = 0.1$
 Continue the pattern for the next five number sentences.

Comparing fractions

	Objectives	Concrete	Pictorial	Abstract	Challenges
Stage 3	<p>Compare and order unit fractions, and fractions with the same denominators</p>	 <p>Using strips of paper, compare these fractions using the $>$, $<$ or $=$ symbols.</p> <p style="text-align: center;"> $\frac{3}{4} \bigcirc \frac{1}{4}$ $\frac{1}{6} \bigcirc \frac{5}{6}$ $\frac{3}{8} \bigcirc \frac{5}{8}$ </p>	<p style="text-align: center;">one whole</p>  <p>Unit fractions: The greater the denominator, the smaller the fraction.</p>  <p style="text-align: center;"> $\frac{1}{10} \bigcirc \frac{1}{4}$ $\frac{1}{3} \bigcirc \frac{1}{6}$ $\frac{1}{5} \bigcirc \frac{1}{4}$ </p>	<p>Place the fractions on the number line.</p>  <p style="text-align: center;"> $\frac{2}{4}$ $\frac{3}{4}$ $\frac{1}{4}$ </p> <p>Complete the missing numerators from these ordered fractions.</p> <p style="text-align: center;"> $\frac{1}{5}$ $\frac{2}{5}$ $\frac{\quad}{5}$ $\frac{4}{5}$ $\frac{5}{5}$ </p>	<p>Only a fraction of each line is shown. The rest is hidden behind the blue screen. Which whole line is the longer? Explain your reasoning.</p>  <p>Make the statement true:</p>  <p>How many ways can you find? Investigate</p> <p>Greater or less than $\frac{1}{2}$? Prove it.</p> 
Stage 5	<p>Compare and order fractions whose denominators are all multiples of the same number</p>	 <p style="text-align: center;">Common denominator = 20</p> 	<p style="text-align: center;"> $\frac{2}{3}$ $\frac{7}{12}$ $\frac{5}{6}$ </p>  <p>Use these models, to compare and order these fractions.</p>	 <p>When comparing and ordering fractions with different denominators, children can write out their times tables to find the common denominator.</p>	<p>Imran put these fractions in order starting with the smallest. Are they in the correct order? Two fifths, three tenths, four twentieths How do you know?</p>

Compare and order fractions, including fractions >1



$$2\frac{1}{5} \bigcirc 2\frac{3}{8}$$

Use the digit cards to complete the statements below:

5 6 3 4

$$\frac{\square}{\square} > \frac{\square}{\square} \quad \frac{\square}{4} < \frac{6}{\square}$$

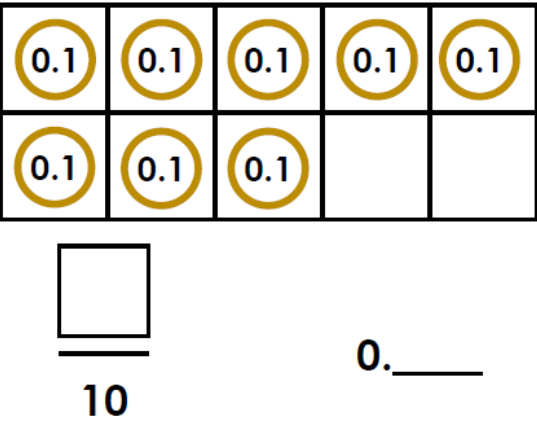
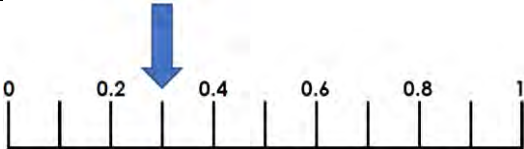
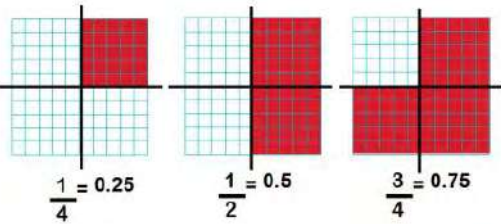
Find three examples of ways you could complete the statement:

$$\frac{\square}{\square} < \frac{\square}{\square}$$

Can one of your ways include an improper fraction?

Give an example of a fraction that is greater than 1.1 and less than 1.5. Now another example that no one will think of. Explain how you know.

Finding fraction and decimal equivalence

	Objectives	Concrete	Pictorial	Abstract	Challenges				
Stage 3	Recognise tenths as fractions and decimals	 <p style="text-align: center;">Use the place value counters and tens frames to complete the fraction and decimal.</p>	 <p>True or false? The arrow shows 0.5</p>	$\frac{3}{10} = \text{---.---}$ $0.7 = \text{---}$	<p>Which is the odd one out?</p> <p>A. $\frac{5}{10}$ B. five tenths</p> <p>C. 0.5 D. <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td style="padding: 2px;">Ones</td><td style="padding: 2px;">Tenths</td></tr><tr><td style="padding: 2px;"></td><td style="padding: 2px; text-align: center;">0.1 0.1 0.1 0.1 0.1 0.1</td></tr></table></p> <p><small>Convince me.</small></p>	Ones	Tenths		0.1 0.1 0.1 0.1 0.1 0.1
Ones	Tenths								
	0.1 0.1 0.1 0.1 0.1 0.1								
Stage 4	Recognise and write decimal equivalents to $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$	Use base 10 to see how many hundredths are needed to cover $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ and this can be used to show the decimal equivalence.		$\frac{1}{4} = 0.25$ $\frac{1}{2} = 0.5$ $\frac{3}{4} = 0.75$	<p>Ordering Put these numbers in the correct order, starting with the smallest.</p> <p>$\frac{1}{4}$ 0.75 $\frac{5}{10}$</p> <p>Explain your thinking</p>				

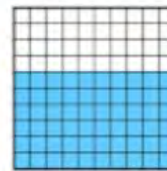
Recognise tenths and hundredths as fractions and decimals



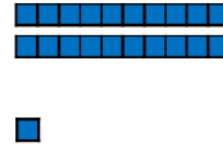
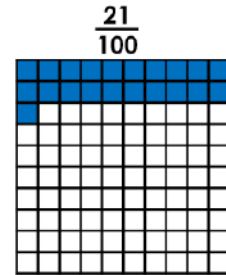
$\frac{1}{10}$ of the chocolate bar = 0.1



0.6
six tenths



0.60
sixty hundredths



True or false?

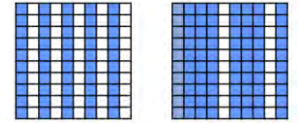
21 hundredths can be partitioned into 2 tenths and 1 hundredth.

$$\frac{1}{10} = 0.1$$

$$\frac{3}{10} = 0.3$$

$$\frac{5}{10} = \frac{1}{2} = 0.5$$

$$\frac{8}{100} = 0.08$$

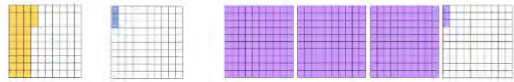


Marcus is using hundred squares to represent one whole and four tenths. Is Marcus correct? Explain your answer

Recognise thousandths, hundredths and tenths as fractions and decimals

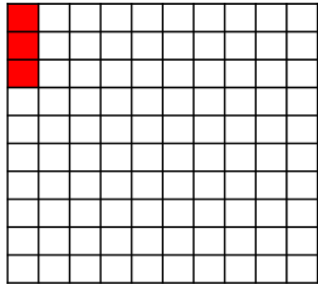
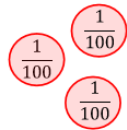
Use the models to record equivalent decimals and fractions.

$0.3 = \frac{3}{10} = \frac{30}{100}$



Use place value counters and base 10 to represent the relationship between fraction hundredths and decimals hundredths 0.01

What fraction is being shown in both representations? Can you convert this in to a decimal?

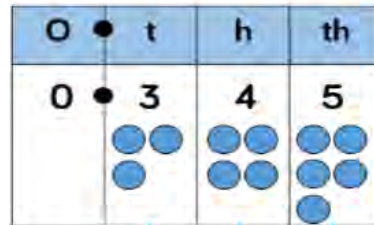


The fraction $\frac{\square}{\square}$ is the same as the decimal \square

Pictorial Representation	Decimal	Decimal - expanded form	Fraction	Fraction - expanded form	In words
	4.251	$4 + 0.2 + 0.05 + 0.001$	$4\frac{251}{1000}$	$4 + \frac{2}{10} + \frac{5}{100} + \frac{1}{1000}$	four ones, two tenths, five hundredths and one thousandth
	4.512				
			$4\frac{25}{1000}$		
				$4 + \frac{5}{10} + \frac{1}{1000}$	

June is converting decimals to thousandths

$0.345 = \frac{\square}{1000}$



$\frac{3}{10} + \frac{4}{100} + \frac{5}{1000} = \frac{300}{1000} + \frac{40}{1000} + \frac{5}{1000} = \frac{345}{1000}$

Use June's method to convert the decimals to thousandths

0.276

0.029

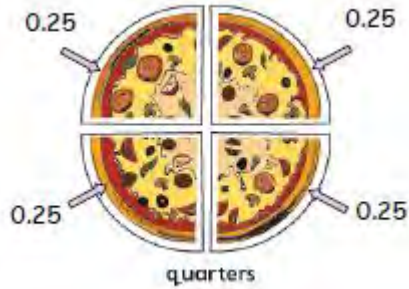
1.286

Another and another Write a fraction with a denominator of one hundred which has a value of more than 0.75?

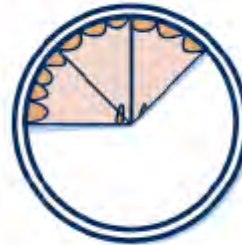
Ordering Put these numbers in the correct order, starting with the largest. $\frac{7}{10}$, 0.73, $\frac{7}{100}$, 0.073, 0.7

Stage 6

Associate a fraction with division and calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction (e.g. 3 / 8)



3 slices of pie 'out of' 8



$$\frac{3}{8}$$

$$\frac{3}{8}$$

3 'out of' 8 is the same as 3 'divided by' 8

$$3 \div 8 = 0.375$$

$$\text{So } \frac{3}{8} = 0.375$$

Another and another


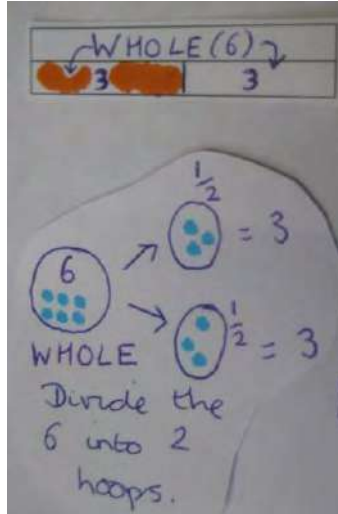

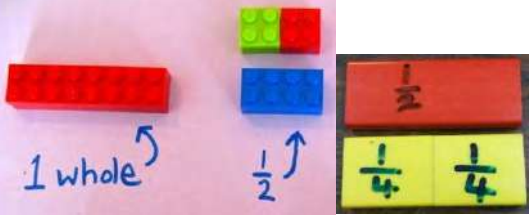
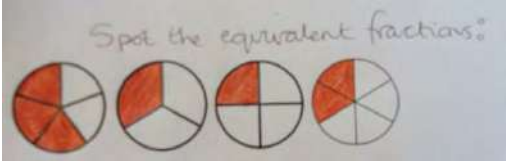

Write a unit fraction which has a value of less than 0.5?

Complete the pattern

$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{4}{8}$
0.375	???	???	???

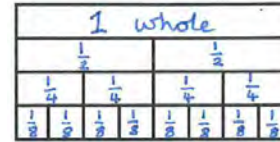
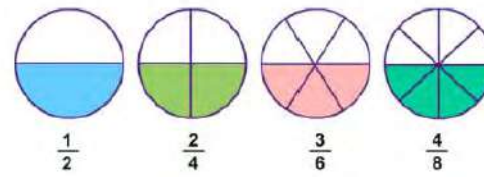
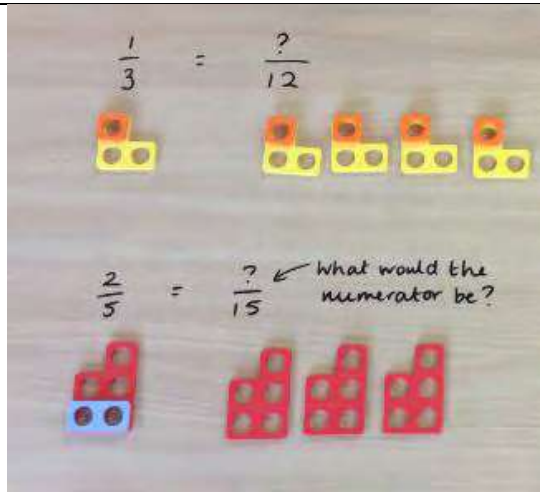
Complete the table.

Equivalent fractions

	Objectives	Concrete	Pictorial	Abstract	Challenges
Stage 2	<p>Write simple fractions for example, $\frac{1}{2}$ of 6 = 3 and recognise the equivalence of $\frac{1}{2}$ and a $\frac{2}{4}$.</p>	<p>Finding a $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ set of objects (less than 20 objects).</p> <p>$\frac{1}{3}$ of 9 cubes = 3</p> 	<p>$\frac{1}{2}$ of 6 cubes = 3</p> 	<p>$\frac{1}{2}$ of 6 cubes = 3</p> <p>$6 \div 2 = 3$</p>	 <p>Leo lost $\frac{1}{2}$ his marbles in a game. This is what he has left. How many did he start with?</p>
Stage 3	<p>Recognise and show, using diagrams, equivalent fractions with small denominators</p>	 <p>How many quarters are equivalent to a half?</p>	<p>Spot the equivalent fractions:</p>  <p>0 $\frac{1}{3}$ $\frac{2}{3}$ 1</p> <p>0 $\frac{1}{6}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$ 1</p>	<p>Images can be used to identify equivalent fractions.</p> <p>$\frac{1}{2} = \frac{\square}{6} = \frac{\square}{12}$</p>	<p>Here is a diagram showing $\frac{1}{2}$</p>  <p>Draw 3 more diagrams showing $\frac{1}{2}$ and write the equivalent fractions.</p>

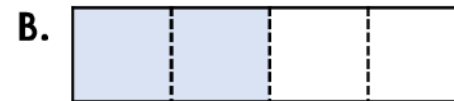
Stage 4

Recognise and show, using diagrams, families of common equivalent fractions



$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$$

Which two fractions are equivalent?



$$\frac{1}{6} \times \square = \frac{4}{24}$$

Odd one out.
Which is the odd one out in each of these trios
 $\frac{3}{4}$ $\frac{9}{12}$ $\frac{4}{6}$
 $\frac{9}{12}$ $\frac{10}{15}$ $\frac{2}{3}$
 Why?

Look at the sequence below:

$$\frac{1}{6}, \frac{2}{12}, \frac{3}{18}$$

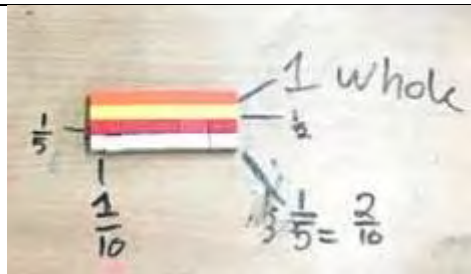
Tara says, The next fraction is $\frac{4}{24}$.

Darren says, The next fraction is $\frac{4}{19}$.

Who is correct? Convince me.

Stage 5

Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths



Children can create their own fraction wall using Cuisenaire.

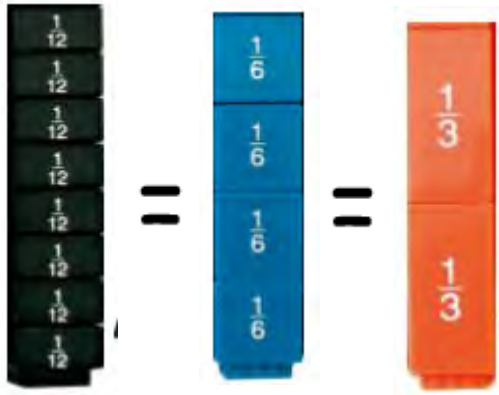


$$\frac{2}{5} \times 4 = \frac{8}{20}$$

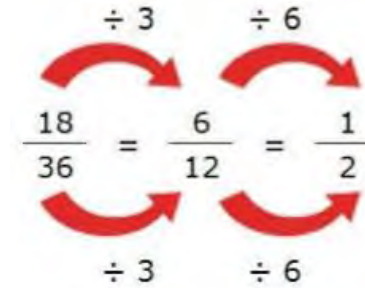
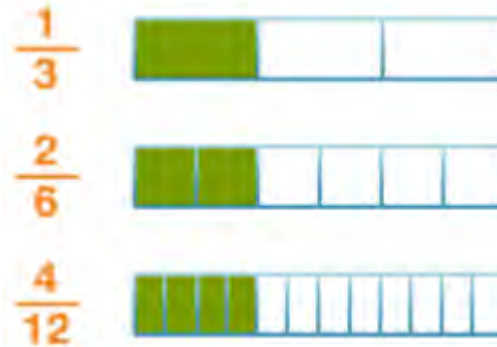
What do you notice?
 Find 30/100 of 200
 Find 3/10 of 200
 What do you notice?
 Can you write any other similar statements?

Stage 6

Use common factors to simplify fractions; use common multiples to express fractions in the same denominator



Children can use fraction walls to create equivalent fractions.



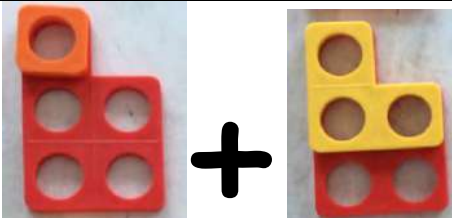
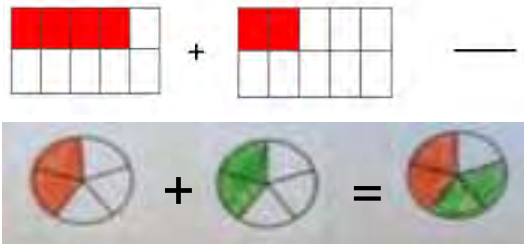
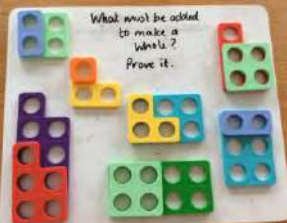

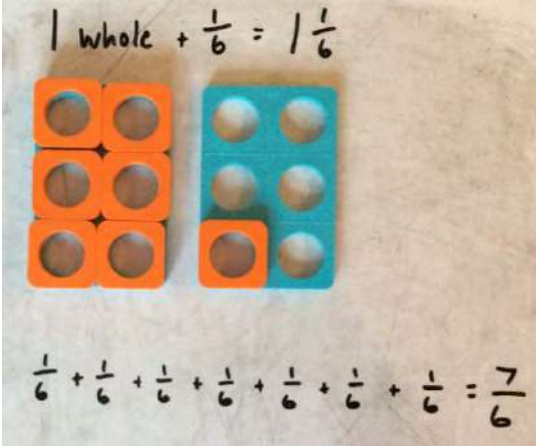
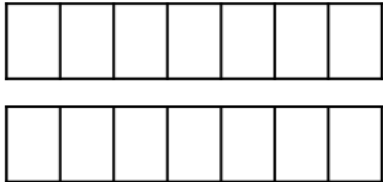
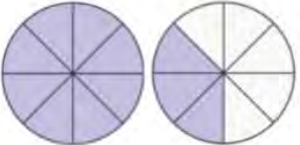
In each number sentence, replace the boxes with different whole numbers less than 20 so that the number sentence is true:

$$\frac{1}{\square} = \frac{3}{\square}$$

$$\frac{\square}{3} = \frac{\square}{12}$$

$$\frac{\square}{\square} = \frac{\square}{\square}$$

Adding and subtracting fractions

	Objectives	Concrete	Pictorial	Abstract	Challenges
Stage 3	Add and subtract fractions with the same denominator within one whole			$\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$ $\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$	  <p>What fractions could you have added together to get this answer?</p>
Stage 4	Add and subtract fractions with the same denominator	<p>1 whole + $\frac{1}{6} = \frac{1}{6}$</p>  <p>$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{7}{6}$</p>	$\frac{4}{7} + \frac{6}{7} = \frac{\square}{\square}$  $\frac{11}{8} - \frac{5}{8} = \square$ 	$\frac{3}{8} + \frac{6}{8} = \frac{\square}{8} + \frac{2}{8} = \frac{\square}{8}$	<p>Use the digit cards to complete this calculation. You can use each card more than once if you wish.</p> <p style="text-align: center;"> 5 7 12 3 </p> $\frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

Stage 5

Add and subtract fractions with the same denominator (see Stages 3 + 4) and multiples of the same number



$$\frac{8}{20} + \frac{5}{20} = \frac{13}{20}$$

$$\frac{2}{5} + \frac{1}{4} = \frac{13}{20}$$

Step 1	Step 2	Step 3
<p>Draw the fraction with the smaller denominator. Shade the fraction.</p> $\frac{1}{4} + \frac{3}{8} =$	<p>Split the model to create the second denominator. Shade the other fraction.</p> $\frac{1}{4} + \frac{3}{8} =$	<p>Now the fractions have the same denominator, you can add the fractions.</p> $\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$
Step 1	Step 2	Step 3
$\frac{1}{3}$	$\frac{4}{12}$	$\frac{1}{3} - \frac{1}{12} = \frac{3}{12}$

$$\frac{4}{12} + \frac{1}{3} = \frac{\square}{\square}$$

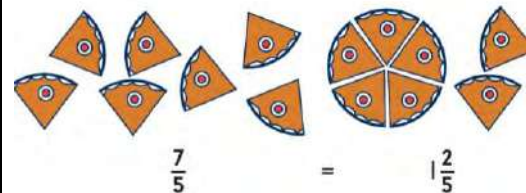
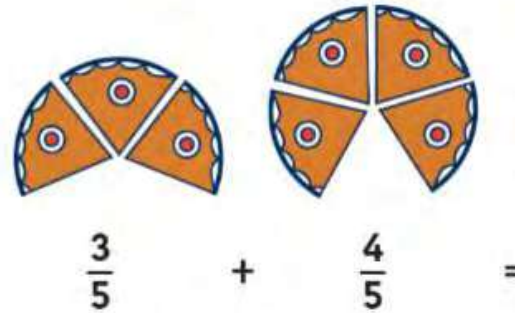
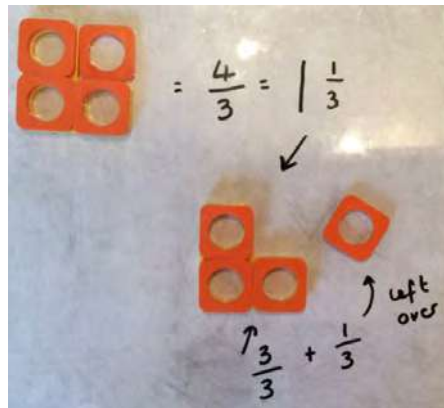
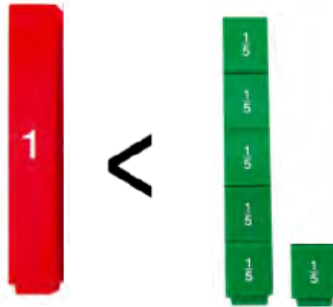
$$\frac{3}{8} + \frac{5}{24} = \frac{\square}{\square}$$

Using the numbers 3, 4, 5 and 6 only once, make this sum have the smallest possible answer:

$$\frac{\square}{\square} + \frac{\square}{\square} =$$

Stage 5

Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number (e.g. $2/5 + 4/5 = 6/5 = 1 1/5$)



$$\frac{7}{2} = 3 \frac{1}{2}$$

because $7 - 2 = 3$ with 1 half left over

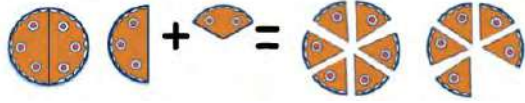
$$2 \frac{1}{3} = \frac{7}{3}$$

because $2 \times 3 = 6$ with 1 third left to add

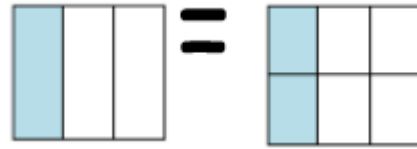
$\frac{3}{4}$ and $\frac{1}{4} = 4/4 = 1$
 $4/4$ and $\frac{1}{4} = 5/4 = 1 \frac{1}{4}$
 $5/4$ and $\frac{1}{4} = 6/4 = 1 \frac{1}{2}$
 Continue the pattern up to the total of 2.

Stage 6

Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions



$$1 \frac{1}{2} = \frac{3}{2} \quad \frac{3}{2} = \frac{9}{6}$$



$$\frac{1}{3} = \frac{2}{6}$$

$$1 \frac{1}{2} + \frac{1}{3} = 1 \frac{5}{6}$$

because $1 \frac{1}{2} = \frac{3}{2}$

$$\frac{3}{2} = \frac{9}{6} \quad \text{and} \quad \frac{1}{3} = \frac{2}{6}$$

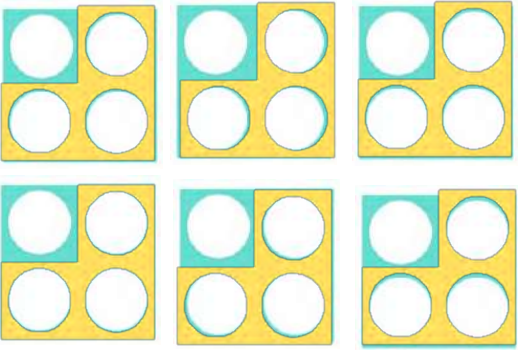
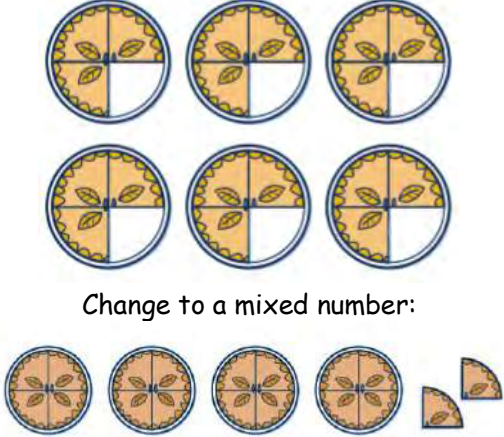
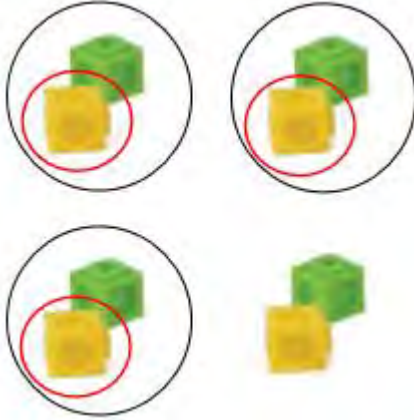

$$\text{so } \frac{9}{6} + \frac{2}{6} = \frac{11}{6} = 1 \frac{5}{6}$$

The answer is $1 \frac{2}{5}$, what is the question?

Another and another

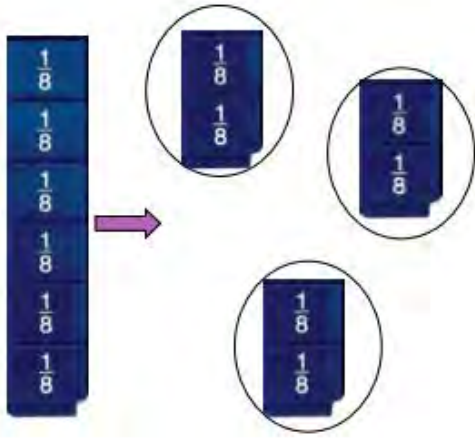
Write down 2 fractions with a total of $3 \frac{4}{5}$ and another, ... and another, ...

Multiplying and dividing fractions

	Objectives	Concrete	Pictorial	Abstract	Challenges
Stage 5	<p>Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams</p>	 <p style="text-align: center; margin-top: 10px;">$6 \text{ lots of } \frac{3}{4}$</p>	 <p style="text-align: center; margin-top: 10px;">Change to a mixed number:</p>	$\frac{3}{4} \times 6 = \frac{18}{4}$ <p style="text-align: center;">Change to a mixed number:</p> $\frac{18}{4} = 4 \frac{2}{4}$	<p>Continue the pattern:</p> <p>$\frac{1}{4} \times 3 = \frac{1}{4} \times 4 = \frac{1}{4} \times 5 =$ Continue the pattern for five more number sentences. How many steps will it take to get to 3?</p> <p>The answer is $2 \frac{1}{4}$, what is the question Give your top tips for multiplying fractions</p>
Stage 6	<p>Multiply simple pairs of proper fractions, writing the answer in its simplest form</p>	 <p style="text-align: center; margin-top: 10px;">$\frac{1}{2} \text{ of } \frac{3}{4}$</p>	<p style="text-align: center; margin-bottom: 10px;">$\frac{1}{2} \text{ of } \frac{3}{4}$</p> 	$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$ <div style="display: flex; align-items: center; justify-content: center;"> <div style="border: 1px solid black; padding: 2px; margin-right: 10px;">1 multiply the numerators</div> <div style="margin-right: 10px;">→</div> <div style="text-align: center;"> \times $\frac{2}{5} \times \frac{5}{6} =$ </div> </div> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;"> <div style="margin-right: 10px;">↑</div> <div style="border: 1px solid black; padding: 2px;">2 multiply the denominators</div> </div> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;"> <div style="margin-right: 10px;">←</div> <div style="border: 1px solid black; padding: 2px;">3 simplify</div> </div> $\frac{10}{30} = \frac{1}{3}$	<p>Can you write your top tips for multiplying proper fractions?</p>

Stage 6

Divide proper fractions by whole numbers



$$\frac{6}{8} \div 3 = \frac{2}{8}$$



$$\frac{1}{2} \div 3 = \frac{1}{6}$$

$$\frac{1}{2} \div 3 = \frac{1}{6}$$

Keep it, change it, flip it!

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

Continue the pattern:

$$\frac{1}{3} \div 2 = \frac{1}{6}$$

$$\frac{1}{6} \div 2 = \frac{1}{12}$$



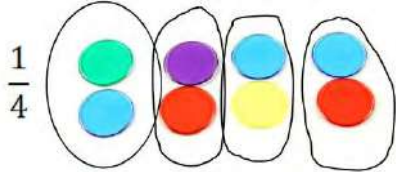

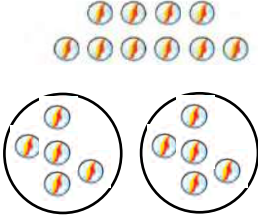
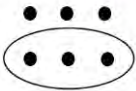

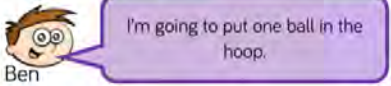
$$\frac{1}{12} \div 2 = \frac{1}{24}$$

What do you notice?

$$\frac{1}{2} \times \frac{1}{4} =$$

The answer is $\frac{1}{8}$,
what is the question
(involving fractions /
operations)

Finding fractions of an amount

	Objectives	Concrete	Pictorial	Abstract	Challenges
Stage 1	<ul style="list-style-type: none"> Find a $\frac{1}{2}$ of a quantity. Find a $\frac{1}{4}$ of a quantity. 	<p>Finding a half and a quarter of an quantity:</p> <p>Find a half of the tower:</p>   <p>Find a quarter of 8 counters:</p> 	<p>Finding a half and a quarter of an quantity:</p> <p>Find half of the amounts.</p>   <p>Beads and marbles can be used as a concrete resource prior to the pictorial representations.</p> <p>Other pictorial representations include drawing circles to represent objects and arrays.</p>	<p>Writing a number sentence alongside the concrete or pictorial representation:</p> <p>$\frac{1}{2}$ of 6 = <input style="width: 30px; height: 20px; border: 1px solid purple;" type="text"/></p> 	<p>Mr. White has asked his class to put one quarter of the balls into the hoop.</p>   <p>Do you agree with Ben?</p>

Find $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity.

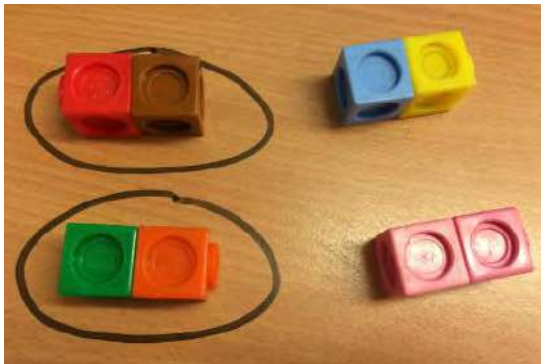
Using numicon to find $\frac{1}{4}$ of 12:



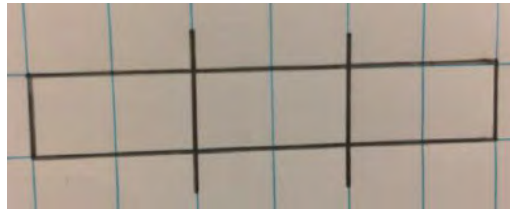
Which numicon piece fits into 12 exactly 4 times - 3 so a $\frac{1}{4}$ of 12 is 3.
To find $\frac{3}{4}$ count up the value of 3 of these 3 pieces = 9.

Find $\frac{2}{4}$ of 8:

First, use the multilink to find $\frac{1}{4}$ (the 8 cubes have been shared into 4 groups). Then, circle 2 of the groups as you want to find $\frac{2}{4}$ and count the total number of cubes.

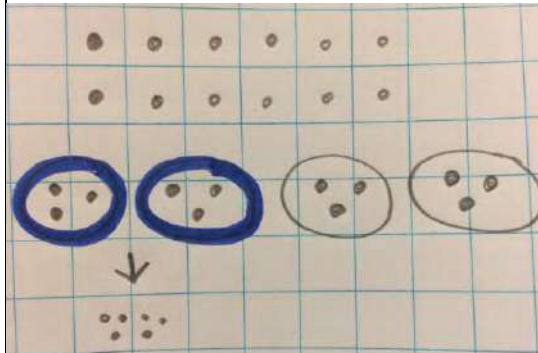


Find $\frac{1}{3}$ of 6:



Find $\frac{2}{4}$ of 12:

First, share the dots between 4 circles to find $\frac{1}{4}$ and then highlight two of the groups as you want to find $\frac{2}{4}$ and count the total number of dots.

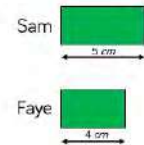


Writing a number sentence alongside the concrete or pictorial representation.

Leo lost $\frac{1}{2}$ his marbles in a game. This is what he has left. How many did he start with?



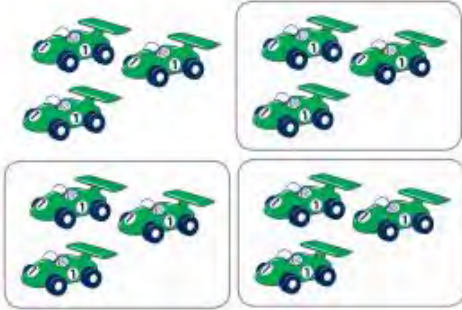
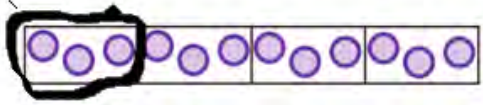



Sam and Faye each have a piece of ribbon that they have cut into quarters.



How long was Sam's whole piece of ribbon?

How long was Faye's whole piece of ribbon?

Stage 3	<p>Find a fractions of a discrete set of objects: (unit fractions and non-unit fractions with small denominators).</p>	<p>For Stages 3 and 4, place value counters can be used to support division when finding fractions of a quantity.</p> <p>For example: Using a bar model and place value counters to find $\frac{1}{4}$ of 84:</p>  <p>To find $\frac{3}{4}$ of 84, the total of the place value counters in 3 of the boxes can be calculated.</p> 	<div style="text-align: center;"> $\frac{3}{4}$  </div> <p>Using a bar model and drawing counters to find a $\frac{1}{4}$ of 12:</p> 	<p>$\frac{1}{5}$ of 15 sweets = $\frac{15}{5}$ 3</p> <p>because $15 \div 5 = 3$.</p> <p>$\frac{2}{5}$ of 15 sweets = $\frac{15 \times 2}{5}$ 6</p> <p>because $15 \div 5 = 3$ and 3×2 is 6.</p>	<p>Kayleigh has 12 chocolates.</p> <p>On Friday, she ate $\frac{1}{4}$ of her chocolates and gave one to her mum.</p> <p>On Saturday, she ate $\frac{1}{2}$ of her remaining chocolates, and gave one to her brother.</p> <p>On Sunday, she ate $\frac{1}{3}$ of her remaining chocolates.</p> <p>How many chocolates does Kayleigh have left?</p>
Stage 4	<p>Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions.</p>	<p>= 63.</p>	<p>$\frac{1}{7}$ of 56 = $56 \div \square$</p>  <p>Once $\frac{1}{7}$ has been calculated, this can be used to find $\frac{2}{7}, \frac{3}{7}$ etc. For $\frac{3}{7}$, 3 boxes can be circled and their totals added up.</p> <p>Dots could be drawn in the boxes to support the division of 56 by 7.</p>	<p>$\frac{2}{3}$ of £18</p> <p>£18 \div 3 = £6</p> <p>£6 \times 2 = £12</p>	<p>How many ways can you make the statement correct?</p> <p>$\frac{2}{9}$ of 81 > $\frac{3}{4}$ of \square</p> <p>Complete the missing number.</p> <p>$\frac{1}{6}$ of \square = 42</p>