Ocklynge Junior School

Progression in Calculation and Written Methods

Written by

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Our Aims:

- By the end of KS2 as many children as possible are equipped with mental, written and calculator methods that they understand and can use correctly.

- When faced with a calculation, children are able to decide when it is best to use a mental, written or calculator method based on the knowledge that they are in control of this choice and are able to carry out all three methods with confidence and have strategies to check accuracy.

- As many children as possible by the end of KS2 can carry out a compact written method for each of the four operations (+,-,x,÷). These written methods will be used for those calculations that cannot be solved easily using a mental calculation method.

- As many children as possible to become fluent in their mathematics through varied and frequent practice with increasingly complex problems over time. Children will become more confident when applying their knowledge to problem solving. They should develop skills in breaking down problems into a series of simpler steps and persevering in seeking solutions.

- As many children as possible should develop reasoning skills by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.

All learning must be underpinned by a secure and appropriate knowledge of number facts. Along with the fluency skills that are needed to carry out the process and judge if it was successful.

The written methods in this document show progression from Stages 1-6 but individual class teachers will decide the appropriate stage for their pupils to access the written method, depending on the ability of the pupils. This may change from year to year.

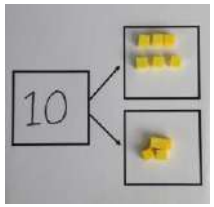
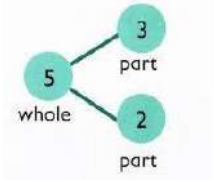
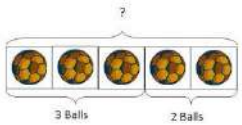
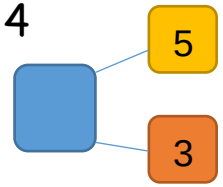
As a whole school we are working toward the concrete, pictorial, abstract approach (CPA) approach. We are also working to deepen children understanding through reasoning skills and problems using documents including White Rose.

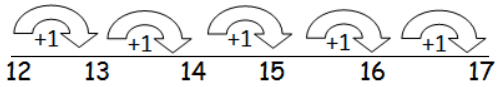
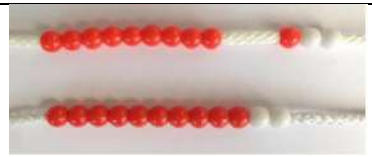
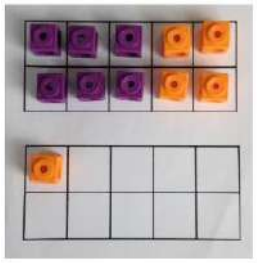
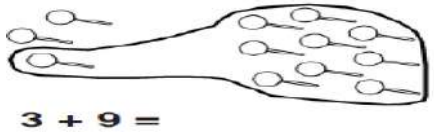
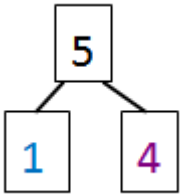
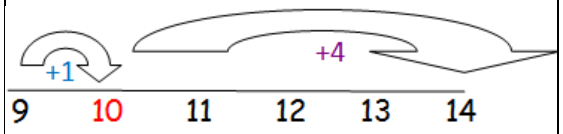
Progression in Addition (+)

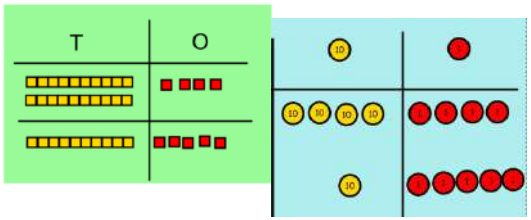
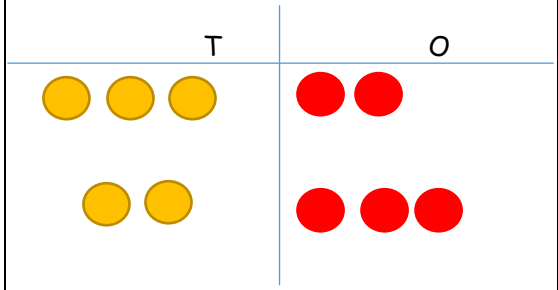
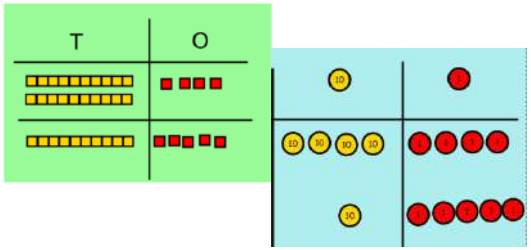
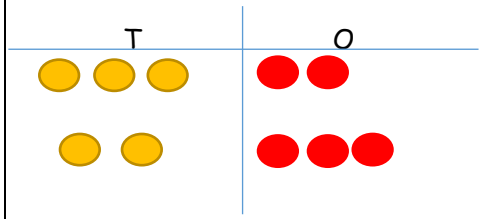
To add successfully, children need to be able to:

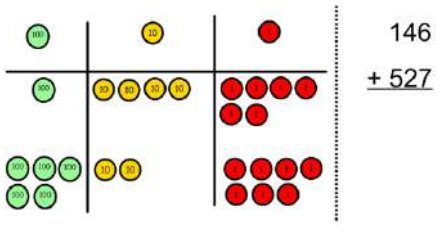
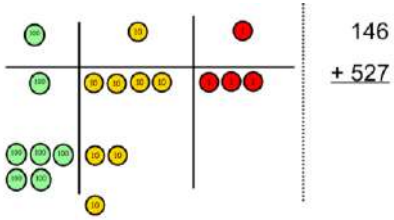
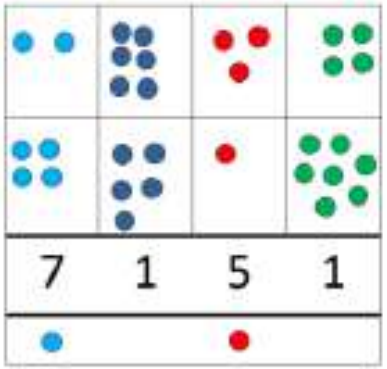
- recall all addition pairs to 9 + 9 and complements in 10;
- add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

	Objective and Strategies	Concrete	Pictorial	Abstract	Fluency
Stage 1	Combining two parts to make a whole: part part whole model.	   Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar. 8 1	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move onto the abstract.	• Counting on from a given number. • Number bonds to 10.

Stages 1 & 2	Starting at the larger number and counting on.	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	**12 + 5 = 17**  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Start with the larger number in your head and count on the smaller number to find your answer.	• Counting on from a given number. • Number bonds to 20. • Add multiples of 1 or 10.
Stage 2	Using number facts to bridge through 10.	 **6 + 5 = 11**  Start with the larger number and use the smaller number to make 10. Remember to add on what's left.	 Use pictures or a number line to bridge through 10.  9 + 5 = 14 	**7 + 4 = 11** If I am at seven, how many more do I need to make 10? How many more do I add on now?	• Counting on from a given number. • Number bonds to 10. • Partitioning numbers. • Add multiples of 1 or 10.

Stages 3 & 4	Column method- partitioning Start with tens and ones and progress through to thousands.	**24 + 15 =** Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. 	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. **32 + 23 =** 	**43 + 56 = 99** **40 + 3** **50 + 6** **90 + 9 = 99** **4 3** **+ 5 6** **9** **+ 9 0** **9 9**	• Counting on from a given number. • Partitioning numbers. • Add multiples of 1, 10, 100 or 1000.
Stages 3 & 4	Column method- no exchanging Start with tens and ones and progress through to thousands.	24 + 15 = Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. 	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. **32 + 23 =** 	**38 + 41 =** 38 41 79	• Counting on from a given number. • Partitioning numbers. • Add multiples of 1, 10, 100 or 1000.

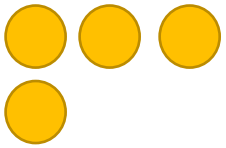
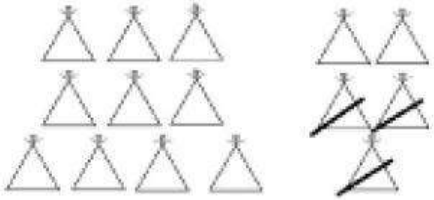
Stages 4, 5 & 6	Column method- exchanging	Make both numbers on a place value grid.  146 + 527 Add up the ones and exchange 10 ones for one 10.  146 + 527 Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.  7 1 5 1	536 + 85 621 1 1 As the children move on, introduce addition of more than two numbers and addition of decimals. Money can be used here. 752 87 + 549 1388 1 1 83.5 +69.7 153.2 1 1	• Partitioning numbers. • Add multiples of 0.1, 1, 10, 100 or 1000. • Add 3 or more single digits efficiently.

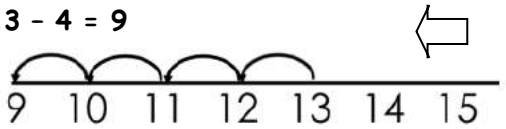
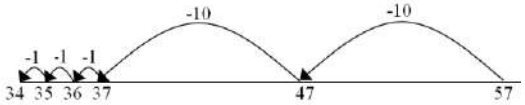
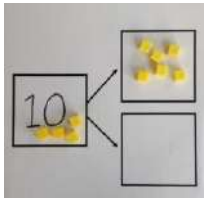
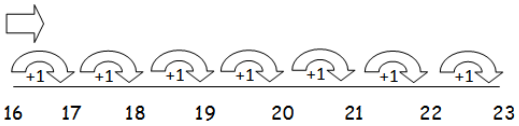
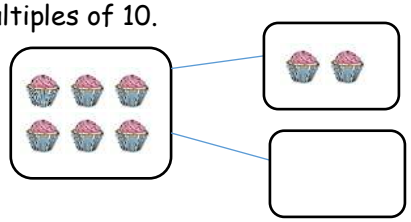
Progression in subtraction (-)

To subtract successfully, children need to be able to:

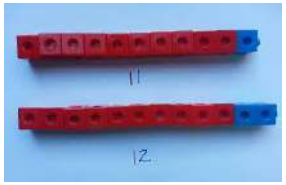
- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as 160 – 70) using the related subtraction fact,16 – 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

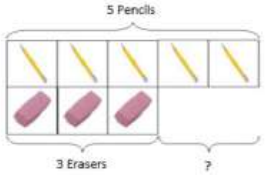
Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

	Objective and Strategies	Concrete	Pictorial	Abstract	Fluency
Stages 1 & 2	Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4 	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	18 – 3 = 15 8 – 2 = 6	• Counting on/back from a given number. • Subtracting multiples of 1.

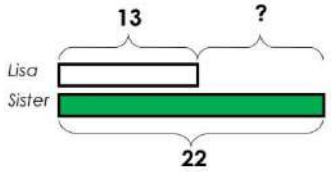
Stages 1 & 2	Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. **13 – 4 = 9**  Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track **13 – 4 = 9**  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This should progress to counting back in multiples of 10.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.	• Counting on/back from a given number. • Subtracting multiples of 1 and 10.
Stages 1 & 2	Counting on	Make the smaller number in your subtraction. Move the beads along your bead string as you count forwards in ones until you reach the larger number. **13 – 4 = 9**  10 - 6 =  If 10 is the whole and 6 is one of the parts. Count on to find the other part.	**23 - 16 = 7**  Start at the smaller number and count on until you reach the larger number. How many jumps did you make? This should progress to counting on in multiples of 10.  Draw a part part whole model to represent the difference.	Put 16 in your head and count on until you reach 23. How many fingers are you holding up?	• Counting on/back from a given number. • Number bonds to 10 and 20. • Using number facts to bridge through 10. • Subtracting multiples of 1 and 10.

<table>
<tr>
<td>Stages 1 & 2</td>
<td>Find the difference</td>
<td>Compare amounts and objects to find the difference.

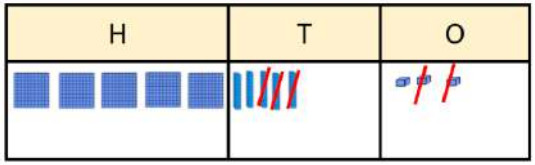
 
Use cubes to build towers or make bars to find the difference

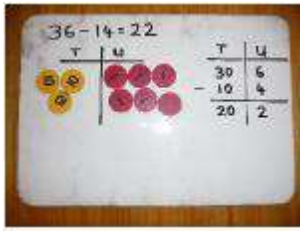
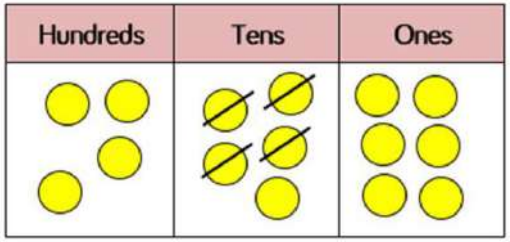
Use basic bar models with items to find the difference</td>
<td>Comparison Bar Models

Lisa is 13 years old. Her sister is 22 years old.
Find the difference in age between them.
 

Draw a bar for each number and count on to find the difference.</td>
<td>Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the numbers of sandwiches.</td>
<td>• Counting on/back from a given number.
• Number bonds to 10 and 20.
• Using number facts to bridge through 10.
• Subtracting multiples of 1 and 10.</td>
</tr>
<tr>
<td>Stages 3 & 4</td>
<td>Column method- partitioning

Start with tens and ones and progress through to thousands.</td>
<td>553 – 32 =
 

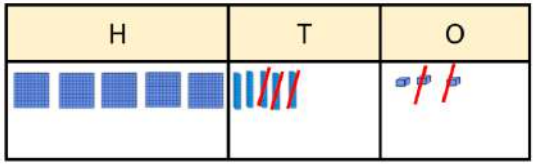
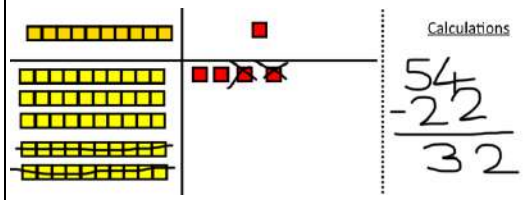
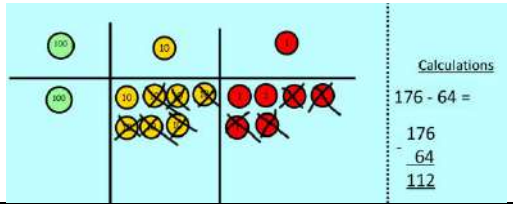
Use base 10 to partition the larger number then take away the smaller number. Begin with the ones.

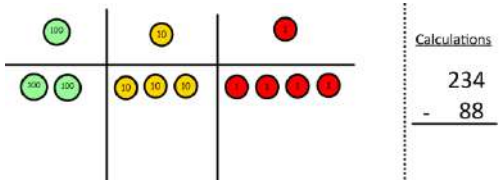
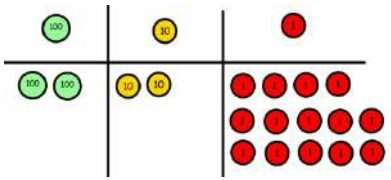
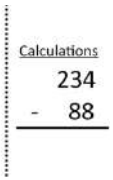
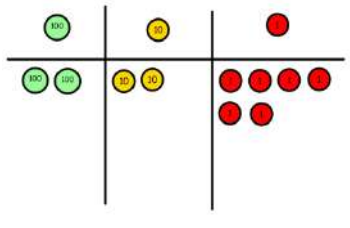
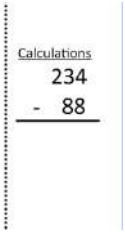
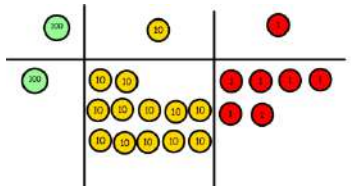
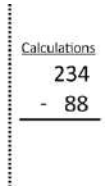
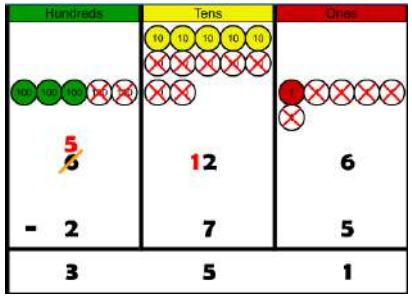
Show how you partition numbers to subtract. Again make the larger number first.
 
</td>
<td>456 – 40 =
 

Draw the counters for your larger number then cross out the smaller number. Begin with the ones.</td>
<td>754 - 23 = 731

700 50 4
– 20 3
700 30 1 = 731

47 – 24 = 23
40 + 7
– 20 + 4
20 + 3</td>
<td>• Partition numbers.
• Subtracting multiples of 1, 10 and 100.
• Subtract digits efficiently.</td>
</tr>
</table>

Stages 3 & 4	Column method- without exchanging	**553 – 32 =**  Use base 10 to partition the larger number then take away the smaller number. Begin with the ones.	 Draw the Base 10 or place value counters alongside the written calculation to help to show working.  	869 - 245 624	• Subtracting multiples of 1, 10 and 100. • Subtract digits efficiently.

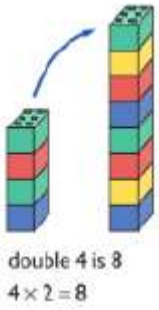
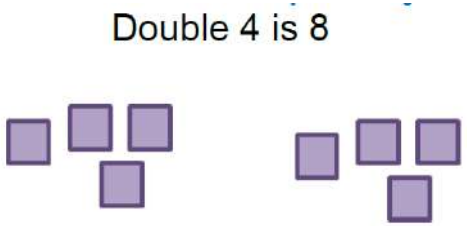
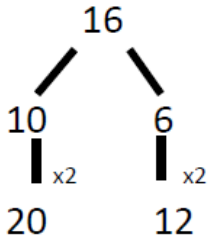
Stages 4, 5 & 6	Column method- exchanging	Use Base 10 to start with before moving on to place value counters. Make the larger number with the place value counters.  Calculations 234 - 88 Start with the ones. Can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.   Calculations 234 - 88 Now I can subtract my ones.   Calculations 234 - 88 Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.   Calculations 234 - 88 Now I can take away eight tens and complete my subtraction	 Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.	6 14 1 7 5 4 - 8 6 6 6 8 As the children move on, they will use the compact method. This will lead to an understanding of subtracting any number including decimals. 5 12 1 2 6 3 . 0 - 2 6 . 5 2 3 6 . 5	• Subtracting multiples of 1, 10 and 100. • Subtract digits efficiently.

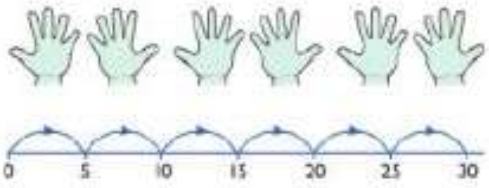
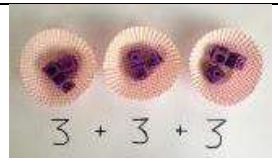
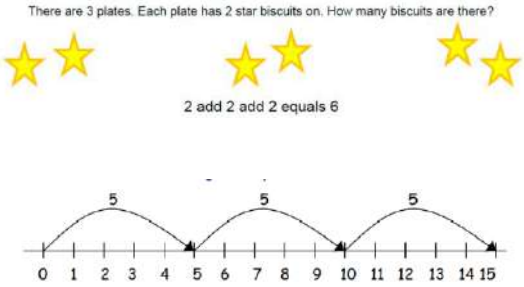
Progression in multiplication x

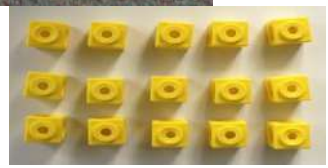
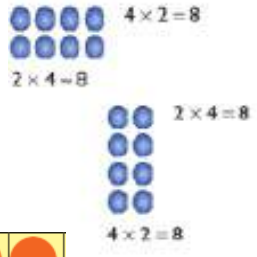
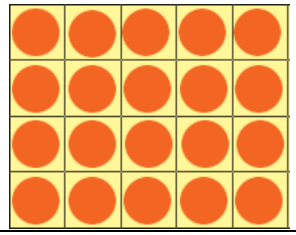
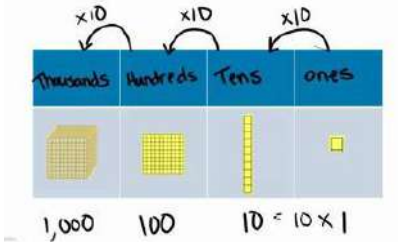
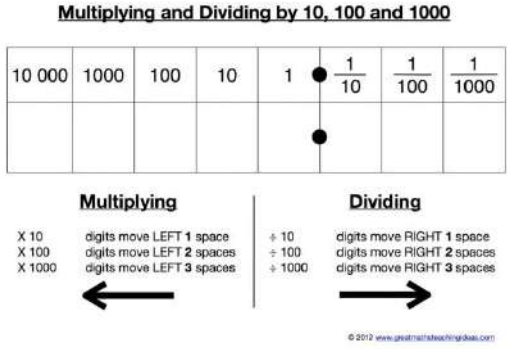
To multiply successfully, children need to be able to:

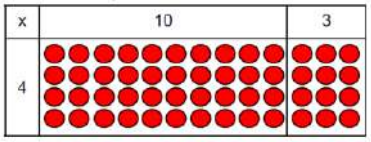
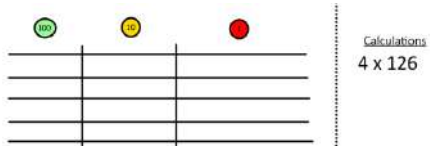
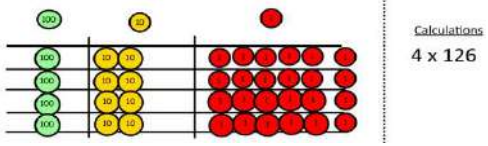
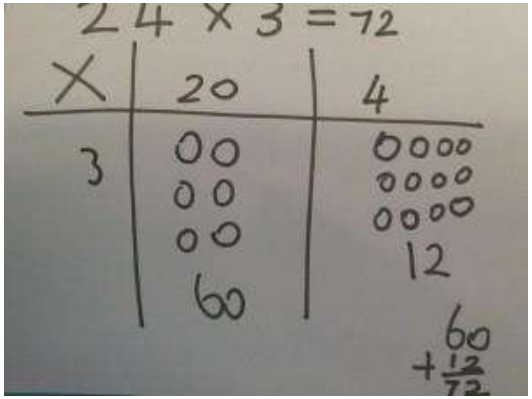
- Recall all multiplication facts to 10 × 10;
- Partition number into multiples of one hundred, ten and one;
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- add combinations of whole numbers using the column method (see above).

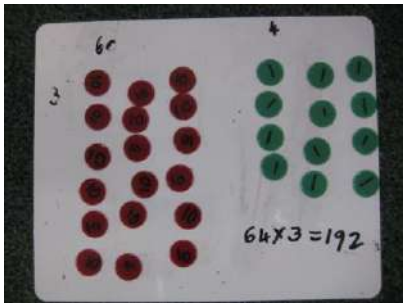
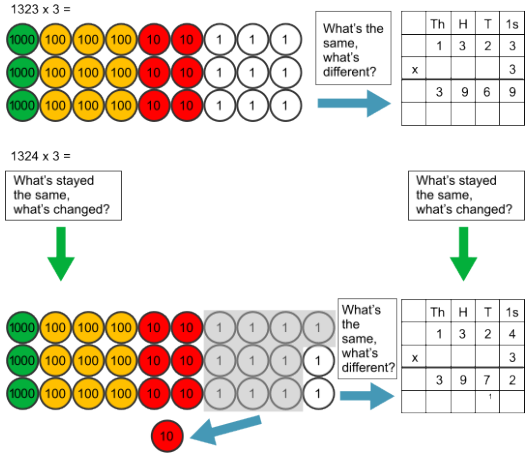
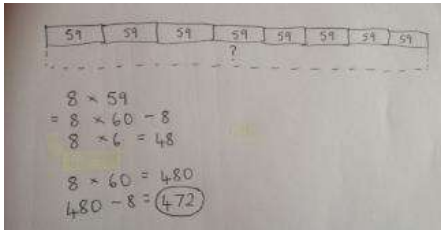
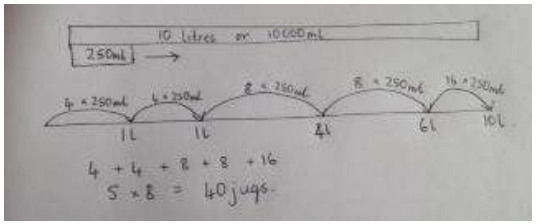
Note: It is important that children's mental methods of calculation are practiced and secured alongside their learning and use of an efficient written method for multiplication.

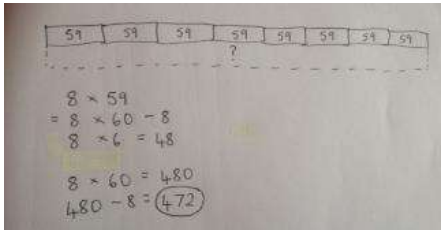
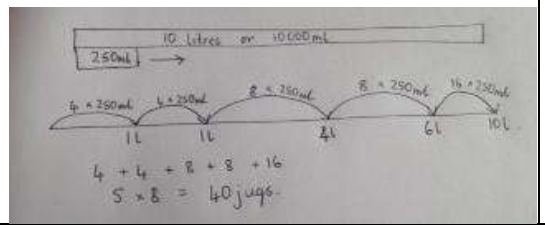
	Objective and Strategies	Concrete	Pictorial	Abstract	Fluency
Stage 1	Doubling	Use practical activities to show how to double a number.  2 + 2 = 4  double 4 is 8 4 × 2 = 8	Draw pictures to show how to double a number.  	Partition a number and then double each part before recombining it back together.  	• Counting in multiples of 2 forwards and backwards. • Multiplication facts for 2 times tables.

Stages 1 & 2	Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30	• Counting in multiples of different amounts forwards and backwards. • Multiplication facts 2, 5 and 10 times tables.
Stages 1 & 2	Repeated addition	   Use different objects to add equal groups.	 5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10	• Counting in multiples of different amounts forwards and backwards. • Repeated addition. • Use knowledge of number patterns to check answers.

Stages 1 & 2	Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.   Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15	• Counting in multiples of different amounts forwards and backwards. • Repeated addition. • Use knowledge of number patterns to check answers.
Stages 3-6	Multiplying by multiples of 10.	Physically jumping into the next column. Use base 10/place value counters and place value boards to demonstrate to move the digits across the columns.  	 	Develop the use of digit shuffle boards and moving your digits through drawings in books. 4 x 10 = 40	• Counting in multiples of 10/100 forwards and backwards. • Multiplication facts for 10 times tables. • Use knowledge of number patterns to check answers.

Stages 3 & 4	Grid Method	Show the link with arrays to first introduce the grid method.  4 rows of 10 4 rows of 3  Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows. Fill each row with 126.  Add up each column, starting with the ones making any exchanges needed. Then you have your answer.	Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | ×	30	5
---	---	---
7	210	35	 **210 + 35 = 245** Moving forward, multiply by a 2 digit number showing the different rows within the grid method. 		10	8
---	---	---
10	100	80
3	30	24 | | X	1000	300	40	2
---	---	---	---	---
10	10000	3000	400	20
8	8000	2400	320	16		• Partition numbers • Multiply by multiples of 10 • Multiplication facts 12 X 12

Stages 4, 5 & 6	Column multiplication – no decimals	Place value counters can be used to support column multiplication.  It is important that they always multiply the ones column first and note down their answer followed by the tens column which they note below.  	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.   	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768 This moves to the more compact method: 482 X 6 2892 4 1 43 X 27 301 +860 1161	• Partition numbers • Multiply by multiples of 10 • Multiplication facts 12 X 12

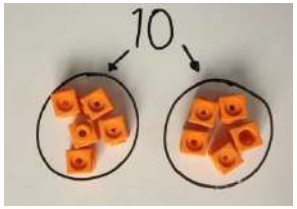
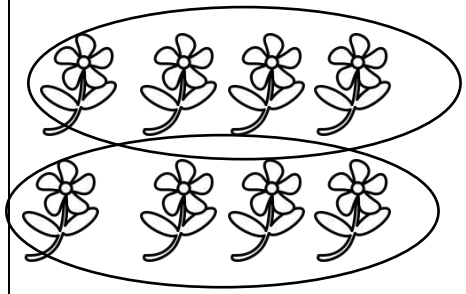
Stages 5 & 6	Column multiplication – decimals	Place value counters can be used to support column multiplication. It is important that they always multiply the right hand column first and note down their answer followed by the next column which they note below.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.   	Multiplying decimals 83.52 X 45 417.60 1 2 1 + 3340.80 1 2 3758.40	• Partition numbers • Multiply by multiples of 10 • Multiplication facts 12 X 12

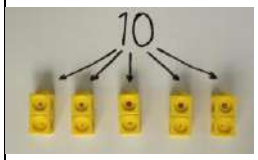
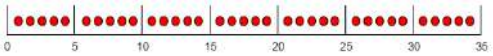
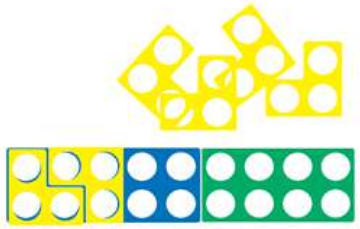
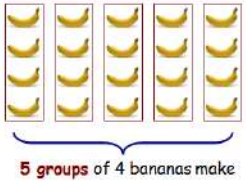
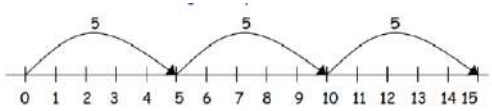
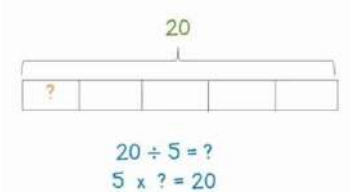
Progression in division ÷

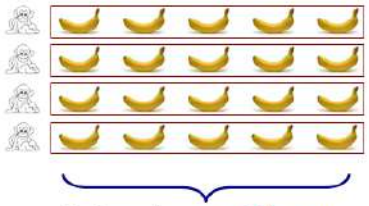
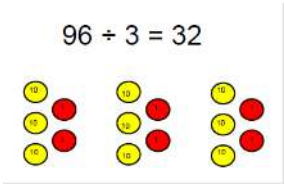
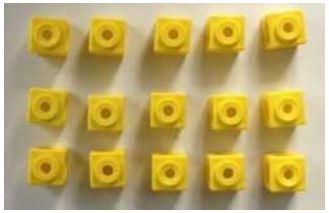
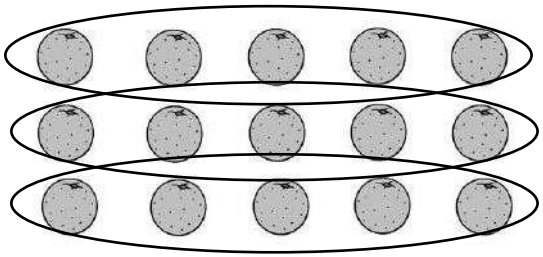
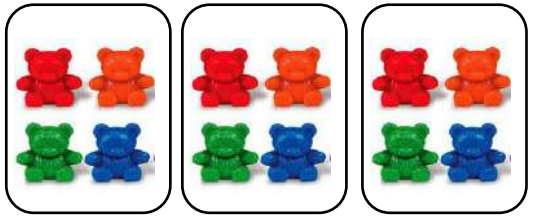
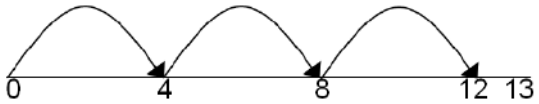
To divide successfully in their heads children need to be able to:

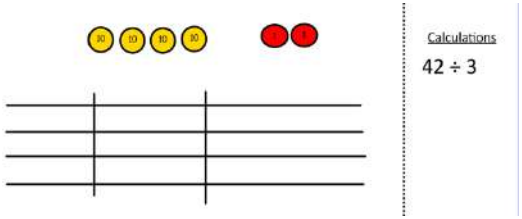
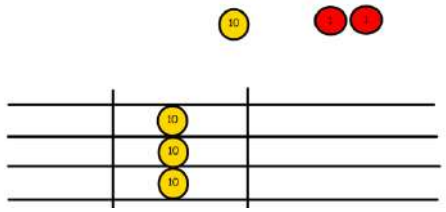
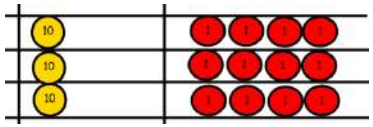
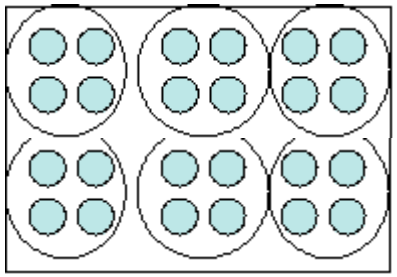
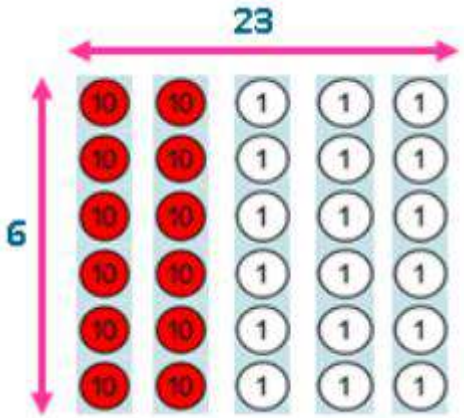
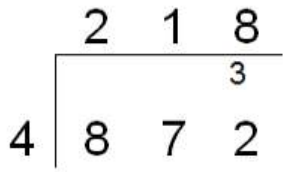
- understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

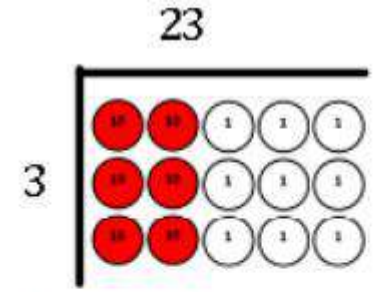
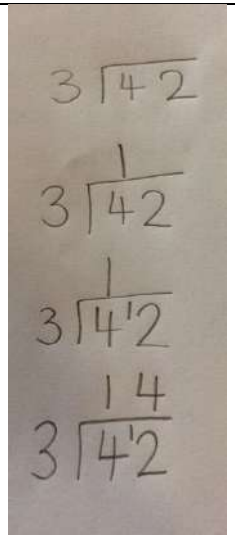
Note: It is important that children's mental methods of calculation are practiced and secured alongside their learning and use of an efficient written method for division.

	Objective and Strategies	Concrete	Pictorial	Abstract	Fluency
Stages 1	Sharing objects into groups	   I have 10 cubes, can you share them equally in 2 groups? **10 ÷ 2 = 5**	Children use pictures or shapes to share quantities.  **8 ÷ 2 = 4**	Share 9 buns between three people. **9 ÷ 3 = 3**	• Cardinal numbers

Stages 1 & 2	Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  **10 ÷ 2 = 5**  **35 ÷ 5 = 7** 18 ÷ 3 = 6  Covering the 18 with the '3' pieces relates to additive approaches to division on a number line. How many bunches (groups) of 4 bananas are there in a pile of 20? 20 ÷ 4 How many 4s in 20?  5 groups of 4 bananas make	 Use a number line to show jumps in groups. The number of jumps equals the number of groups. **15 ÷ 5 = 3**   Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.	**28 ÷ 7 = 4** Divide 28 into 7 groups. How many are in each group?	• Understand division as repeated subtraction

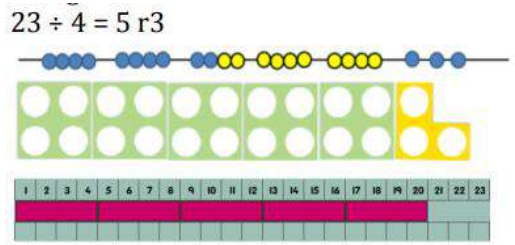
Stages 1 & 2	Division as sharing	**20** bananas need to be shared equally between **4** monkeys. How many does each monkey get? **20 ÷ 4**  Each monkey gets **5** bananas.	This is an example of how place value counters could be used then drawn into the children's books. 	**28 ÷ 7 = 4** 28 grapes are shared between 7 children. How many grapes would each child have?	• Understand division as repeated subtraction
Stages 1 & 2	Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. **15 ÷ 3 = 5** **5 x 3 = 15** **15 ÷ 5 = 3** **3 x 5 = 15**	 Draw an array and circle the groups to make a division calculations: **15 ÷ 5 = 3**	Find the inverse of multiplication and division sentences by creating four linking number sentences. **7 x 4 = 28** **4 x 7 = 28** **28 ÷ 7 = 4** **28 ÷ 4 = 7**	• Multiplications facts and related division facts (inverse).
Stages 2 & 3	Division with a remainder	Divide objects between groups and see how much is left over   **14 ÷ 3 = 3r2**	**1. Number line** Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  **13 ÷ 4 = 3r1** **2. Draw an array** Draw dots and group them to divide an amount and clearly show a remainder. 	Complete written divisions and show the remainder using r. 	• Multiplications facts and related division facts (inverse).

Stages 3, 4, 5 & 6	Short division	**42 ÷ 3 = 14** Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look at how much is in 1 group so 42 ÷ 3 = 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups. **24 ÷ 4 = 6**  Encourage them to move towards counting in multiples to divide more efficiently. **138 ÷ 6 = 23** You could also draw out the place value counters which have been divided into 6 rows to make 138.  	Begin with divisions that divide equally with no remainder.  	• Multiplications facts and related division facts (inverse). • Estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92

Stages 3, 4, 5 & 6	Short division	**42 ÷ 3 = 14** In addition, you could use a bead string to demonstrate this. First, use the beads to represent your 4 tens.  Next, show how many groups of 3 you can make.  As you have a 10 remaining, you need to exchange this for 10 ones which you combine with the ones that are already there.  Then, group your ones. 	 69 ÷ 3 = 23 Check using multiplication inverse: 23 x 3 = 69	 This recording would be done alongside the use of a bead string.	• Multiplications facts and related division facts (inverse). • Estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92

Stages 4, 5 & 6

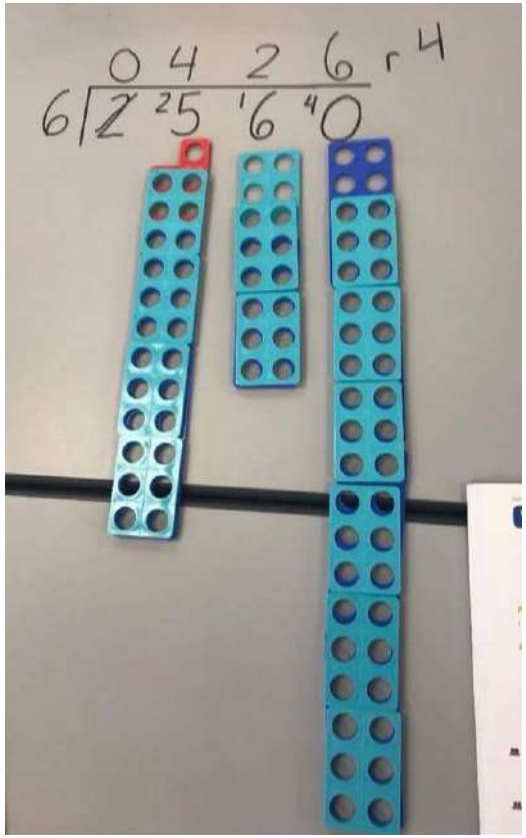
Short division with remainders

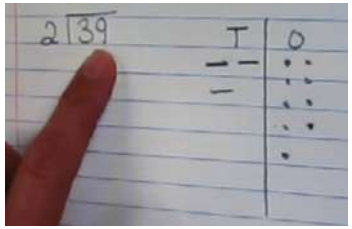


2560 ÷ 6 = 426 r 4

Use concrete resources like Numicon alongside the written method.

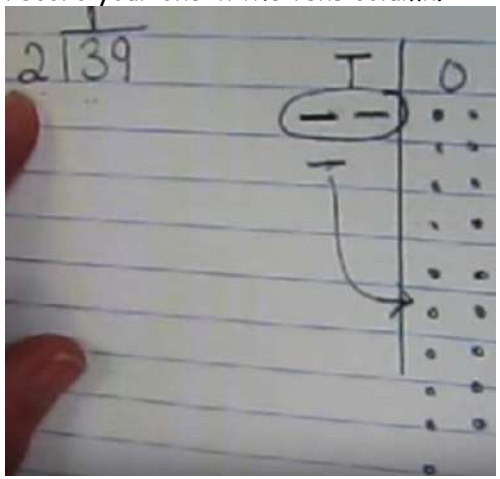
Create the numbers using Numicon then put the Numicon plates you are dividing by on top of the Numicon that represent each number.



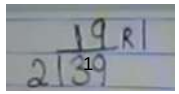


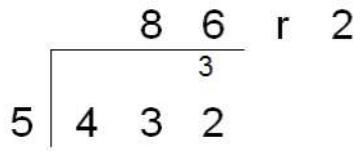
Children could draw base 10 to support their calculation.

First, look at how many groups of 2 you have from the 3 tens and circle. Then record your one in the tens column.



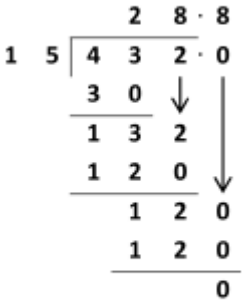
Exchange the remaining ten for ten ones and see how many groups of 2 you can make.





In Stages 5 and 6, move onto representing remainders in different ways (decimals, fractions and number).

- Multiplications facts and related division facts (inverse).
- Estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92

Stage 6				Long division: 432 ÷ 15 becomes  List multiples of 15 to support long division. Answer: 28·8 To help children to understand this method, show how to answer the same calculation using short division alongside the long division question.	• Multiplications facts and related division facts (inverse). • Estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92 • Identify common factors, common multiples and prime numbers.